SHOCK WAVE PROPAGATION IN A DOUBLE EXPLOSION IN A GAS WITH COUNTERPRESSURE

É. I. Andriankin and N. N. Myagkov

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Problems of shock propagation with energy liberation by one point explosion in a gas were investigated in sufficient detail [1-3]. In examining two explosions [4], which can occur at different points of space and be initiated at a different time in the general case, a number of new dimensionless parameters appears which can, be varied and thus substantially alter the nature of the nonlinear shock interaction and the one or more resultant discontinuities being formed. One limit case of such a problem is the explosion of a charge above a plane surface (which is equivalent to the simultaneous explosion of two identical charges separated by a distance equal to twice the height above the surface), and it has been studied theoretically and experimentally by many authors [5-7]. The parameter on which the solution of this problem depends is the dimensionless height of the charge. In this paper another limit case is examined, that of the problem of a double explosion when the point explosions occur at the same point of space but at different times. The solution will then depend on two control parameters, the energy ratio $\lambda^\circ = E_2^\circ/E_1^\circ$ for the first and second explosions, respectively, and the delay time between the explosions t_0, where the case $E_1^\circ + E_2^\circ = \text{const}$ is of special interest.

1. Formulation of the problem is analogous to that considered earlier by the authors [8], however, with the substantial distinction that now counterpressure P_0 by the medium ahead of the first shock is taken into account. We consider the first point explosion to occur at the time $t = -t_0$ at the point $r = 0$ and the second to occur at the time $t = 0$ at the same point of space. The density of the unperturbed gas ρ_0, the viscosity, and the heat conduction are not taken into account. The gas flow behind the discontinuities is adiabatic, subject to the equation of state of a perfect gas $\varepsilon = P/(\gamma - 1)\rho$ (ε is the specific internal energy) with the adiabatic index $\gamma = 1.4$.

Taken as time and distance scales are $t^0 = r^0/(P_0/\rho_0)^{1/2}$, $r^0 = (E_1^0/P_0\alpha_0)^{1/\nu}$ where α_0 is the self-similar constant, and plane ($\nu = 1$) or spherical ($\nu = 3$) symmetry is considered. We introduce the dimensionless variables $t = t't^0$, $r = r'r^0$, $\rho = \rho'\rho_0$, $P = P'P_0$, $v = v'r^0/t^0$. The delay time is $t_0' = t_0/t^\circ$. The dimensionless quantities are here denoted by a prime, which we later omit by considering that we deal just with dimensionless quantities. As $t_0 \to 0$ the solution of the problem of a double explosion goes over into the solution of a problem of one explosion with the energy $E_1^\circ + E_2^\circ$.

The initial system of gas dynamics equations was solved numerically by the S. K. Godunov method in an implicit difference scheme with a subsequent conversion based in the integral conservation laws [9] with extraction of the flow singularities. Two shocks following each other and a high entropy zone at the center of the explosions are formed in a double point explosion. Hence, the high-entropy zone at the center as well and the first and second shock discontinuities were extracted in the computation until they merged, and after the merger, the front of the resultant shock was extracted. Provided in the program was a change in the number of cells in the two computation domains as the flow evolved and the dimensions of these domains changed. The initial conditions for the first explosion were given by the numerical solution of the self-similar problem of a strong explosion [1] and for the second explosion from the solution of the linearized problem [8] for pressures on the order of 150-200 on the fronts. The accuracy of the computations was checked by the mass and energy conservation laws, where the error did not exceed 0.6% for the spherical case and 2-3% for the plane case.

2. Results of the numerical solution are represented in Figs. 1-4. As the parameters t_0 and λ° change, the nature of the wave interaction changes. The dependence of P_*, the amplitude of the second shock, is shown in Fig. 1 for spherical symmetry and $\lambda^\circ = 1$, and after merger, the amplitude of the resultant shock, as a function of the distance to the center of the explosions. The curves 1-3 correspond to the delay $t_0 = 0.02, 0.08, 0.12$; curve 4

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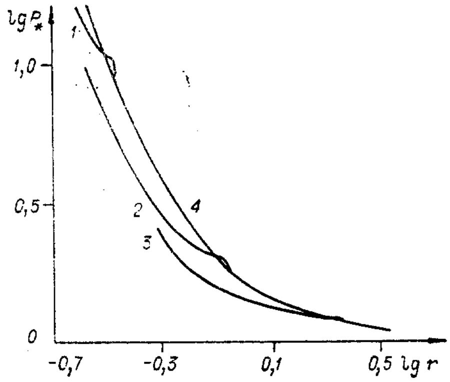

Fig. 1

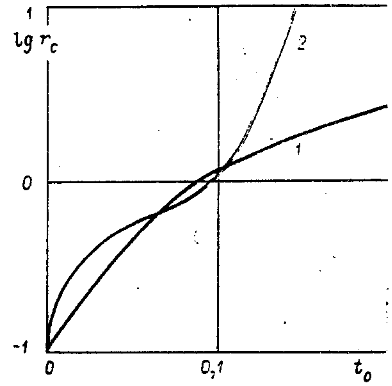

Fig. 2

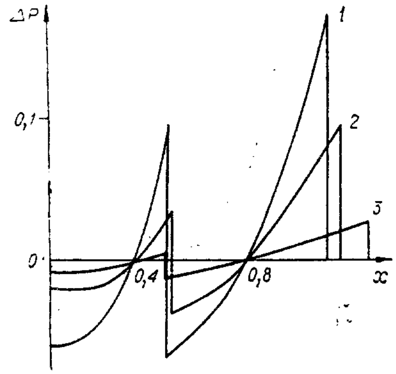

Fig. 3

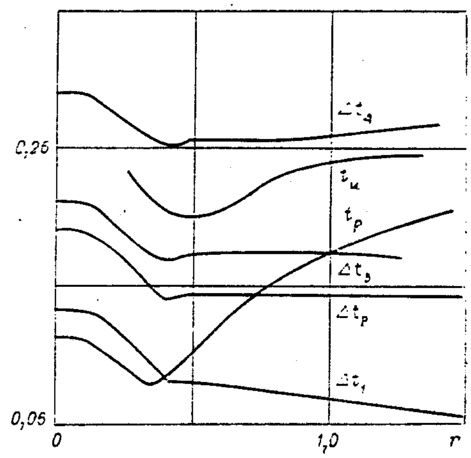

Fig. 4

to the amplitude of the pressure during the explosion of one charge with energy $2E_1^\circ$ ($t_0 = 0$). It is seen that after the merger of the discontinuities of the double explosion (the distance of the merger can be determined from Fig. 2), the amplitude of the resultant wave rapidly emerges on the asymptotic of the single explosion of energy $2E_1^\circ$. Computations show that the impulse of the positive excess pressure phase also possesses an analogous property.

An important characteristic for the one-dimensional shocks being propagated is the distance for merger of their discontinuity fronts $r_c = r(\lambda^0, t_0)$; however, in a number of cases the C_+-characteristic or the shock discontinuity cannot overtake the shock front preceding them. This occurs with the characteristic it turns out to be in the negative phase of the shock since, as is known, the point of transition from the positive to the negative phase at large distances from the site of the explosion can be a limit characteristic: All C_+-characteristics in the negative phase do not intersect it, no matter how long prolonged. The dependence of the distance for the shock fronts to merge on the delay time t_0 for a double explosion with $\lambda^0 = 1$ is displayed in Fig. 2 for plane (curve 1) and spherical (curve 2) symmetry. In the plane case the second shock always overtakes the first; it is seen in the figure that in this case the distance to merger r_c grows monotonically in the time range t_0 under consideration. For spherical symmetry, the dependence $r_c(t_0)$ has an inflection, then rising abruptly the merger distance tends to infinity for finite t_0: The second shock discontinuity cannot overtake the first.

The evolution of shocks for the case when the second discontinuity does not overtake the first, shown in Fig. 3, is typical. Figure 3 is constructed for values $\lambda^0 = 0.6$, $t_0 = 0.4$, of the double explosion parameters; the running coordinate $x = r - \sqrt{\gamma}t$ is laid off along the abscissa axis, and the excess pressure $\Delta P = P - 1$ along the ordinate. The configuration 1 corresponds to the time $t \simeq 0.667$, starting with $t \simeq 1.4$ (configuration 2), the velocity of the second discontinuity becomes less than the speed of sound in a certain Lagrange particle at the time of going over from the positive to the negative phase, and consequently, the duration of the negative phase of the first wave starts to grow. In a certain time interval the flow between the discontinuities recalls an N-wave profile (configuration 3 corresponds to $t \simeq 5.76$). Involving the theory of the second approximation in nonlinear accoustics [10] in the consideration, it is easy to see that further wave evolution at considerable distance will result in the amplitude of the second discontinuity in Fig. 3 becoming less than zero; the negative phases of both waves merge and one shock is formed with a jump of the gasdynamic quantities in the negative phase.

It is perfectly evident that for sufficiently large λ° ($E_2^\circ > E_1^\circ$), the merger distance will be finite for arbitrary but finite t_0. Hence, for the case of spherical explosions critical values of the parameters λ_*° and $(t_0)_*$ can be extracted which will define the domain $t_0 > (t_0)_*$. $\lambda^\circ < \lambda_*^\circ$ depends on the λ°, t_0 plane ($(t_0)_*$ depends on λ°), yielding those values of the parameters t_0 and λ° for which the second discontinuity cannot overtake the first.

The existence of a critical delay time $(t_0)_*$ is due to interaction of the second discontinuity with the rarefaction phase of the first wave; thus, for the delays $t_0 \geqslant 0.12$ the negative excess pressure phase, and for the delays $t_0 \geqslant 0.25$, the negative velocity phase succeed in appearing prior to initiation of the second explosion; consequently, for such t_0 the second shock discontinuity is propagated a certain distance in the rarefaction phase, which causes its additional attenuation. The distance of second discontinuity interaction with the rarefaction phases can be determined from the intersection of the curves $\Delta t(r)$ with $t_p(r)$ and $t_u(r)$ in Fig. 4, which is constructed for $\lambda^\circ = 1$ and $\nu = 3$. The $\Delta t(r)$ in Fig. 4 is the time interval between the arrival of the first and second discontinuities at a given Euler coordinate r, and t_p and t_u are the durations of the positive excess pressure and velocity phases of the first shock, respectively. The subscripts 1, 2, 3, 4 of Δt correspond to the delay times $t_0 = \Delta t(0) =$ 0.14; 0.20; 0.22; 0.30. In three of the four cases displayed in Fig. 4, the second wave interacts just with the negative excess pressure phase; hence, the curve Δt reaches the curve t_p sufficiently rapidly. In the case $t_0 = 0.30$ the second wave is propagated over the negative excess pressure and velocity phases, and in this case the attenuation of the second discontinuity is substantial, the curves Δt_4, t_u in Fig. 4 are almost parallel, and they intersect only at the distance $r \geqslant 10$. Starting from the second approximation of nonlinear acoustics, it can be shown that the impossibility of the second discontinuity overtaking the first in a weak shock wave asymptotic is related to the nonpositivity of the total impulse (or area) of the negative phase of the first wave and the positive phase of the second wave at distances where this approximation is applicable (in Fig. 3, for instance).

It is seen from Fig. 4 that the shock discontinuities in a certain range of distances (we denote it by Δr) are capable, starting with a distance r_k ($r_k = 0.5$ in Fig. 4), of forming a stable configuration of discontinuities, i.e., a configuration in which the time interval between discontinuities Δt remians constant (we denote it by $T = T(t_0, \lambda^0)$), and that for the delay $t_0 = 0.30$ $\Delta r \simeq 0.2$ and $T \simeq 0.268$, for $t_0 \simeq 0.2$ $\Delta r \geqslant 1.0$ and $T = 0.155$, etc. The formation of a stable configuration is here manifest regardless of whether the first overtakes the second discontinuity on the asymptotic or not. Let us emphasize that in the evolution of a stable configuration of discontinuities, the profile behind the discontinuities changes while the duration of the time interval between the discontinuities remains constant. The condition for constancy of T in Δr is that the velocities of the discontinuities be equal at every point Δr, and this requires, in turn, that the amplitude and profile of the second shock be matched completely uniquely with the amplitude and profile of the first wave at the point r_k, starting with which the stable configuration of discontinuities is observed. It is sufficiently interesting that the second wave "readjusts" under the first wave during evolution from $r = 0$ to $r = r_k$ in a sufficiently broad delay range t_0, in such a way that this becomes possible.

By increasing t_0 considerably, independent wave propagation can be achieved for a double explosion in at least a certain range of distances; hence, for $\lambda^\circ \sim 1$ a stable configuration of discontinuities can generally be obtained with an arbitrarily large time interval $T \sim t_0$. In contradiction to this, a diminution of t_0 does not imply a proportionate diminution in T; for a given λ° it is impossible to obtain a stable configuration with an arbitrarily small dimensionless time T (the dimensional time interval can be made arbitrary by selecting E_1°, P_0, ρ_0 in the necessary manner), which is due to the nature of second wave interaction with the compression and rarefaction phases behind the first shock. For $\lambda^\circ = 1$, $\nu = 3$ the minimal value of the duration $T \simeq 0.9$ corresponds to the delay $t_0 \simeq 0.14$, and the excess pressure on the discontinuities does not exceed ~1.7 at the point $r = r_k$. The approximation of nonlinear

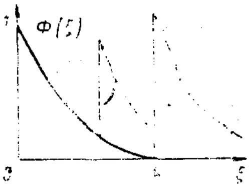

Fig. 5

acoustics is already applicable to velocities of wave discontinuities with such amplitudes. In this approximation the condition that the velocities of the discontinuities of both waves be equal is written if the form $v_+^{(1)} = v_+^{(2)} + v_-^{(2)}$, where $v_+^{(1)}$ is the velocity behind the first discontinuity, $v_+^{(2)}$ is the velocity behind the second discontinuity, and $v_-^{(2)}$ is the velocity ahead of the second discontinuity.

3. Let us consider the possibility of the formation of a stable configuration of discontinuities with arbitrary profiles by starting from the nonlinear acoustics approximation (the theory of the second approximation). In this approximation the wave propagation and interaction along a selected characteristic direction without taking account of dissipation and dispersion processes is described by the solution for Riemann waves in a second approximation [10]. Hence, if the profile at $r = r_k$ is considered as initial, the values of the velocities and the amplitude of the discontinuity at the distances $r > r_k$ can be set in a one-to-one correspondence with a continuous series of points on the profile behind and ahead of the discontinuity. This is seen from the Riemann solution in the second approximation for both shock discontinuities [10]:

$$u_-^{(1)} = 0,\; u_+^{(1)} = \Phi_1\left(\xi_p^{(1)} + z u_+^{(1)}\right),$$
$$u_-^{(2)} = \Phi_1\left(\xi_p^{(2)} + z u_-^{(2)}\right),\; u_+^{(2)} = \Phi_2\left(\xi_p^{(2)} + z u_+^{(2)}\right). \tag{3.1}$$

For the spherical geometry the notation has the form $u = vr/r_K$, $z = [(\gamma + 1)/2\gamma] r_K \ln(r/r_K)$, $\xi = t - (r - r_K)/\sqrt{\gamma}$, ξ_p are coordinates of the discontinuities. The time reference is selected so that $t = 0$ corresponds to the arrival of the first discontinuity at the point $r = r_k (z = 0)$. We also assume that the characteristic wavelength is $l \ll r_K$. The functions Φ_1 and Φ_2 express the initial profiles of the first and second shocks at $z = 0$:

$$u = 0,\ \xi < 0;\ u = \Phi_1(\xi),\ 0 \leqslant \xi < T;\ u = \Phi_2(\xi),$$
$$T \leqslant \xi < \infty. \tag{3.2}$$

The initial coordinate of the first discontinuity is $\xi_p^{(1)} = 0$, and of the second is $\xi_p^{(2)} = T$.

Therefore, the possibility of the propagation of a stable configuration of discontinuities on the segment Δr is associated with the consistency of the initial ($r = r_k$) wave profiles (3.2) on sections adjoining the discontinuities up to points of the profiles corresponding to the distance $r_K + \Delta r$.

Let $\xi = f_1(u)$ and $\xi = f_2(u)$ be functions inverse to Φ_1 and Φ_2. We consider the profile $f_1(u)$ given. Then from the condition of constancy of the time interval between discontinuities $\xi_p^{(2)} - \xi_p^{(1)} = T$ (equality of the velocities of the discontinuities) we obtain a system of equations from (3.1) and (3.2) that determine the second wave profile $f_2(u)$ such that for $z > 0$ a stable configuration of discontinuities would be formed with the time interval T:

$$f_2(u_2) = f_1(u_1 - u_2) + (2u_2 - u_1)z,\quad f_1(u_1 - u_2) = f_1(u_1) - u_2 z + T,$$
$$z_1(u_1) = \frac{2}{u_1^2}\int_{\Phi_1(0)}^{u_1} p f_1'(p)\,dp,\quad T \geqslant f_1(u_1/2) - f_1(u_1),\quad \Phi_2(T) = \Phi_1(0) - \Phi_1(T). \tag{3.3}$$

For a linear or concave profile $\Phi_1(\xi)$ of the first wave ($\Phi_1''(\xi) \geqslant 0$ and $\Phi_1'(\xi) < 0$ for $\xi \in (0, T)$) displayed in Fig. 5, second wave profiles are shown schematically that have been obtained as the solution of the system (3.3) with different T. At the point $\xi = T$ the initial second wave profile can be both convex and concave, depending on the specific function $\Phi_1(\xi)$, but the first derivative $\Phi_2'(T)$ remains less than zero. The solution for the profile $\Phi_2(\xi)$ has a characteristic ambiguity. Only the upper part of the profile $\Phi_2(T) \geqslant \Phi_2(\xi) > u_m$ (u_m is defined as the maximum point of the function $f_2(u)$, $f_2'(u_m) = 0$) corresponds to the real wave process, and to eliminate the ambiguity it is necessary to replace the lower part of the profile $f_2(u_m) < \xi < \infty$ so that the funciton $\Phi_2(\xi)$ would be defined uniquely at every point $\xi > T$ (the lower section of the profile satisfying the single-valuedness condition is shown in Fig. 5 by dashes). The section of the profile $\Phi_2(T) \geqslant \Phi_2(\xi) \geqslant u_m$ for given f_1 and T corresponds to the maximum distance Δz at which the waves will be propagated with constant duration between the fronts T; $\Delta z = \Delta z(u_m, T)$ is found from the solution of (3.3). In the general case the solution of (3.3) can only be found numerically. For the case of a linear first wave profile $\Phi_1(\xi) = 1 - \xi/\alpha$, the solution is written in the form

$$f_2(u) = \alpha(1 - u) - \alpha\sqrt{T/\alpha u} + 2T,\ u_m < u < \Phi_2(T) = T/\alpha,$$

$$u_m = \left(\frac{1}{2}\sqrt{T/\alpha}\right)^{2/3},\quad \Delta z = (2T\sqrt{\alpha})^{2/3} - \alpha,\ T > \alpha/2.$$

It is interesting to note that the second wave profile turns out to be nonlinear. Examination of the problem within the framework of the theory of the second approximation affords the possibility of the existence of weak shocks Φ_1 and Φ_2 capable of forming a stable configuration of discontinuities, regardless of the method of obtaining such waves. The consistency of the wave profiles for the formation of stable configurations depends on the method of their initiation and evolution up to the point r_k. For the double explosion considered here the duration T and the interval Δr depend exclusively on λ^0 and t_0, and can be obtained only because of a numerical solution of the problem.

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